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Semiotik als Theorie gradativer Relationalität

1. In Toth (2019) hatten wir uns gefragt, was denn eine Relation zur semiotischen Relation macht. Nun hatte zwar Bense (1981, S. 17 ff.) das Zeichen als triadische Relation über Primzeichen oder Zeichenzahlen eingeführt

$Z = (1, 2, 3)$

und die Isomorphie der Zeichenzahlen mit den Peanozahlen bereits in Bense (1975, S. 167 ff.) aufgezeigt, allein, während die Peanozahlen die Folge

$P = (1, 2, 3, ..., n)$

bilden, bilden die Zeichenzahlen eine ganz andere Folge

$Z = (1, ((1, 2), (1, 2, 3)), (1, ((1, 2), ((1, 2, 3), (1, 2, 3, 4))), ...),$

insofern

$2 := (1 \rightarrow 2)$

$3 := (1 \rightarrow 2) \rightarrow (1 \rightarrow 2 \rightarrow 3)$, usw.

definiert sind. Vgl. dazu Bense (1979, S. 53):

ZR(M, O, I) =

ZR(M, M → O, M → O → I) =

ZR(mon. Rel., dyad. Rel., triad. Rel.)

ZR(.1., .2., .3.) =

1.1	1.2	1.3	1.1	1.2	1.3	1.1	1.2	1.3
			2.1	2.2	2.3	2.1	2.2	2.3
						3.1	3.2	3.3

„Mit dieser Notation wird endgültig deutlich, daß Repräsentation auf Semiotizität und Semiotizität auf Gradation der Relationalität beruht“ (Bense 1979, S. 53). MAN DARF SOMIT EINE SEMIOTISCHE RELATION ALS EINE GRADATIVE RELATION DEFINIEREN. Und da somit gilt

$Z = (1, 2, 3) = (1 \rightarrow ((1 \rightarrow 2) \rightarrow (1 \rightarrow 2 \rightarrow 3))),$

ist Z also eine Relation, die sich selbst und alle ihre Teilrelationen enthält. Als selbstreflexive Relation kann sie allerdings nur mittels einer Mengentheorie beschrieben werden, für die das Fundierungsaxiom nicht gilt (vgl. Aczel 1988).

2. Im folgenden gehen wir von der Menge der 3! = 6 Permutationen von Z = (1, 2, 3) aus

$P_1 = (1, 2, 3)$

$P_2 = (1, 3, 2)$

$P_3 = (2, 1, 3)$

$P_4 = (2, 3, 1)$

$P_5 = (3, 1, 2)$

$P_6 = (3, 2, 1)$.

Wegen

$Z = (1, 2, 3) = (1 \rightarrow ((1 \rightarrow 2) \rightarrow (1 \rightarrow 2 \rightarrow 3)))$

bekommen wir sofort

$Z_1 = (1 \rightarrow ((1 \rightarrow 2) \rightarrow (1 \rightarrow 2 \rightarrow 3)))$

$Z_2 = (1 \rightarrow ((1 \rightarrow 2 \rightarrow 3) \rightarrow (1 \rightarrow 2)))$

$Z_3 = ((1 \rightarrow 2) \rightarrow (1 \rightarrow (1 \rightarrow 2 \rightarrow 3)))$

$Z_4 = ((1 \rightarrow 2) \rightarrow ((1 \rightarrow 2 \rightarrow 3) \rightarrow 1))$

$Z_5 = ((1 \rightarrow 2 \rightarrow 3) \rightarrow (1 \rightarrow (1 \rightarrow 2)))$

$Z_6 = ((1 \rightarrow 2 \rightarrow 3) \rightarrow ((1 \rightarrow 2) \rightarrow 1))$.

Wir können nun Gradationsstufen einführen, da die oben definierten Abbildungen für 2 und 3 natürlich unendlich oft iterierbar sind. In jedem Z^0_1 bezeichnet das Subskript die Permutation aus (Z_1 ... Z_6) und das Superskript die Gradationsstufe. Als gradative 0-Stufe wird die Permutation vor Einsetzung festgesetzt.

2.1. Z_1

$Z^0{}_1 = (1 \to ((1 \to 2) \to (1 \to 2 \to 3)))$

$Z^1{}_1 = (1 \to ((1 \to 2) \to (1 \to ((1 \to 2) \to (1 \to 2 \to 3)))))$

$Z^2{}_1 = (1 \to ((1 \to (1 \to 2)) \to (1 \to ((1 \to (1 \to 2)) \to (1 \to (1 \to 2) \to (1 \to 2 \to 3)))))$

$Z^3{}_1 = (1 \to ((1 \to (1 \to (1 \to 2))) \to (1 \to ((1 \to (1 \to (1 \to 2))) \to (1 \to (1 \to (1 \to 2)) \to (1 \to (1 \to 2) \to (1 \to 2 \to 3)))))))$

$Z^4{}_1 = (1 \to ((1 \to (1 \to (1 \to (1 \to 2)))) \to (1 \to ((1 \to (1 \to (1 \to (1 \to 2)))) \to (1 \to (1 \to (1 \to (1 \to 2))) \to (1 \to (1 \to (1 \to 2)) \to (1 \to (1 \to 2) \to (1 \to 2 \to 3))))))))$

$Z^5{}_1 = (1 \to ((1 \to (1 \to (1 \to (1 \to (1 \to 2))))) \to (1 \to ((1 \to (1 \to (1 \to (1 \to (1 \to 2))))) \to (1 \to (1 \to (1 \to (1 \to (1 \to 2)))) \to (1 \to (1 \to (1 \to (1 \to 2))) \to (1 \to (1 \to (1 \to 2)) \to (1 \to (1 \to 2) \to (1 \to 2 \to 3)))))))))$

...

2.2. Z_2

$Z^0{}_2 = (1 \to ((1 \to 2 \to 3) \to (1 \to 2)))$

$Z^1{}_2 = (1 \to ((1 \to (1 \to 2) \to (1 \to 2 \to 3)) \to (1 \to (1 \to 2))))$

$Z^2{}_2 = (1 \to ((1 \to (1 \to (1 \to 2)) \to (1 \to (1 \to 2) \to (1 \to 2 \to 3))) \to (1 \to (1 \to (1 \to 2)))))$

$Z^3{}_2 = (1 \to ((1 \to (1 \to (1 \to (1 \to 2))) \to (1 \to (1 \to (1 \to 2)) \to (1 \to (1 \to 2) \to (1 \to 2 \to 3)))) \to (1 \to (1 \to (1 \to (1 \to 2))))))$

$Z^4{}_2 = (1 \to ((1 \to (1 \to (1 \to (1 \to (1 \to 2)))) \to (1 \to (1 \to (1 \to (1 \to 2))) \to (1 \to (1 \to (1 \to 2)) \to (1 \to (1 \to 2) \to (1 \to 2 \to 3))))) \to (1 \to (1 \to (1 \to (1 \to (1 \to 2)))))))$

$Z^5{}_2 = (1 \to ((1 \to (1 \to (1 \to (1 \to (1 \to (1 \to 2))))) \to (1 \to (1 \to (1 \to (1 \to (1 \to 2)))) \to (1 \to (1 \to (1 \to (1 \to 2))) \to (1 \to (1 \to (1 \to 2)) \to (1 \to (1 \to 2) \to (1 \to 2 \to 3)))))) \to (1 \to (1 \to (1 \to (1 \to (1 \to (1 \to 2))))))))$

...

2.3. Z_3

$Z^0{}_3 = ((1 \to 2) \to (1 \to (1 \to 2 \to 3)))$

$Z^1{}_3 = ((1 \to (1 \to 2)) \to (1 \to (1 \to (1 \to 2) \to (1 \to 2 \to 3))))$

$Z^2{}_3 = ((1 \to (1 \to (1 \to 2))) \to (1 \to (1 \to (1 \to (1 \to 2)) \to (1 \to (1 \to 2) \to (1 \to 2 \to 3)))))$

$Z^3{}_3 = ((1 \to (1 \to (1 \to (1 \to 2)))) \to (1 \to (1 \to (1 \to (1 \to (1 \to 2))) \to (1 \to (1 \to (1 \to 2)) \to (1 \to (1 \to 2) \to (1 \to 2 \to 3))))))$

$Z^4{}_3 = ((1 \to (1 \to (1 \to (1 \to (1 \to 2))))) \to (1 \to (1 \to (1 \to (1 \to (1 \to (1 \to 2)))) \to (1 \to (1 \to (1 \to (1 \to 2))) \to (1 \to (1 \to (1 \to 2)) \to (1 \to (1 \to 2) \to (1 \to 2 \to 3)))))))$

$Z^5{}_3 = ((1 \to (1 \to (1 \to (1 \to (1 \to (1 \to 2)))))) \to (1 \to (1 \to (1 \to (1 \to (1 \to (1 \to (1 \to 2)))))) \to (1 \to (1 \to (1 \to (1 \to (1 \to 2)))) \to (1 \to (1 \to (1 \to (1 \to 2))) \to (1 \to (1 \to (1 \to 2)) \to (1 \to (1 \to 2) \to (1 \to 2 \to 3))))))))$

...

2.4. Z_4

$Z^0{}_4 = ((1 \to 2) \to ((1 \to 2 \to 3) \to 1))$

$Z^1{}_4 = ((1 \to (1 \to 2)) \to ((1 \to (1 \to 2) \to (1 \to 2 \to 3)) \to 1))$

$Z^2{}_4 = ((1 \to (1 \to (1 \to 2))) \to ((1 \to (1 \to (1 \to 2)) \to (1 \to (1 \to 2) \to (1 \to 2 \to 3))) \to 1))$

$Z^3{}_4 = ((1 \to (1 \to (1 \to (1 \to 2)))) \to ((1 \to (1 \to (1 \to (1 \to 2))) \to (1 \to (1 \to (1 \to 2)) \to (1 \to (1 \to 2) \to (1 \to 2 \to 3)))) \to 1))$

$Z^4{}_4 = ((1 \to (1 \to (1 \to (1 \to (1 \to 2))))) \to ((1 \to (1 \to (1 \to (1 \to (1 \to 2)))) \to (1 \to (1 \to (1 \to (1 \to 2))) \to (1 \to (1 \to (1 \to 2)) \to (1 \to (1 \to 2) \to (1 \to 2 \to 3))))) \to 1))$

$Z^5{}_4 = ((1 \to (1 \to (1 \to (1 \to (1 \to (1 \to 2)))))) \to ((1 \to (1 \to (1 \to (1 \to (1 \to (1 \to 2))))) \to (1 \to (1 \to (1 \to (1 \to (1 \to 2)))) \to (1 \to (1 \to (1 \to (1 \to 2))) \to (1 \to (1 \to (1 \to 2)) \to (1 \to (1 \to 2) \to (1 \to 2 \to 3)))))) \to 1))$

...

2.5. Z_5

$Z^0{}_5 = ((1 \to 2 \to 3) \to (1 \to (1 \to 2)))$

$Z^1{}_5 = ((1 \to (1 \to 2) \to (1 \to 2 \to 3)) \to (1 \to (1 \to (1 \to 2))))$

$Z^2{}_5 = ((1 \to (1 \to (1 \to 2)) \to (1 \to (1 \to 2) \to (1 \to 2 \to 3))) \to (1 \to (1 \to (1 \to (1 \to 2)))))$

$Z^3{}_5 = ((1 \to (1 \to (1 \to (1 \to 2))) \to (1 \to (1 \to (1 \to 2)) \to (1 \to (1 \to 2) \to (1 \to 2 \to 3)))) \to (1 \to (1 \to (1 \to (1 \to (1 \to 2))))))$

$Z^4{}_5 = ((1 \to (1 \to (1 \to (1 \to (1 \to 2)))) \to (1 \to (1 \to (1 \to (1 \to 2))) \to (1 \to (1 \to (1 \to 2)) \to (1 \to (1 \to 2) \to (1 \to 2 \to 3))))) \to (1 \to (1 \to (1 \to (1 \to (1 \to (1 \to 2)))))))$

$Z^5{}_5 = ((1 \to (1 \to (1 \to (1 \to (1 \to (1 \to 2))))) \to (1 \to (1 \to (1 \to (1 \to (1 \to 2)))) \to (1 \to (1 \to (1 \to (1 \to 2))) \to (1 \to (1 \to (1 \to 2)) \to (1 \to (1 \to 2) \to (1 \to 2 \to 3)))))) \to (1 \to (1 \to (1 \to (1 \to (1 \to (1 \to (1 \to 2))))))))$

...

2.6. Z_6

$Z^0{}_6 = ((1 \to 2 \to 3) \to ((1 \to 2) \to 1))$

$Z^1{}_6 = ((1 \to (1 \to 2) \to (1 \to 2 \to 3)) \to ((1 \to (1 \to 2)) \to 1))$

$Z^2{}_6 = ((1 \to (1 \to (1 \to 2)) \to (1 \to (1 \to 2) \to (1 \to 2 \to 3))) \to ((1 \to (1 \to (1 \to 2))) \to 1))$

$Z^3{}_6 = ((1 \to (1 \to (1 \to (1 \to 2))) \to (1 \to (1 \to (1 \to 2)) \to (1 \to (1 \to 2) \to (1 \to 2 \to 3)))) \to ((1 \to (1 \to (1 \to (1 \to 2)))) \to 1))$

$Z^4{}_6 = ((1 \to (1 \to (1 \to (1 \to (1 \to 2)))) \to (1 \to (1 \to (1 \to (1 \to 2))) \to (1 \to (1 \to (1 \to 2)) \to (1 \to (1 \to 2) \to (1 \to 2 \to 3))))) \to ((1 \to (1 \to (1 \to (1 \to (1 \to 2))))) \to 1))$

$Z^5{}_6 = ((1 \to (1 \to (1 \to (1 \to (1 \to (1 \to 2))))) \to (1 \to (1 \to (1 \to (1 \to (1 \to 2)))) \to (1 \to (1 \to (1 \to (1 \to 2))) \to (1 \to (1 \to (1 \to 2)) \to (1 \to (1 \to 2) \to (1 \to 2 \to 3)))))) \to ((1 \to (1 \to (1 \to (1 \to (1 \to (1 \to 2)))))) \to 1))$

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